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Two General Methods for Calculating the Aircraft Equivalent Singly Vulnerable Area

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Two general methods are proposed to calculate the aircraft equivalent singly vulnerable area. One is called the Monte Carlo simulation based method, and the other is called the aircraft multiple hit vulnerability based method. The Monte Carlo simulation method compares all the aircraft's unique existent states to a "model of filling boxes with balls," and by randomly and uniformly sampling the threat hit locations, the expected number of hits on the vulnerable area required to kill an aircraft E_{AV} can be attained. In the second method, E_{AV} is obtained using the cumulated kill probability of an aircraft subjected to multiple threat hits. After getting the E_{AV}, the aircraft equivalent singly vulnerable area can be calculated. Four examples demonstrate the correctness and feasibility of the two general methods. Analysis shows that the developed methods overcome the shortcomings of currently used methods in that these methods can deal with the calculation of an aircraft equivalent singly vulnerable area for the case in which 1) the aircraft may have several sets of multiply vulnerable components, and the multiply vulnerable components of each set do not always have the same vulnerable area, and 2) the aircraft vulnerable components can overlap in any arbitrary manor.

I. Introduction

AIRCRAFT combat survivability is defined as the capability of an aircraft to avoid or withstand a man-made hostile environment [1]. Survivability is composed of two focus areas: susceptibility and vulnerability. The vulnerability of the aircraft for a particular threat aspect is usually expressed as the probability the aircraft is killed given a random (uniformly distributed) hit anywhere on the presented area of the aircraft, that is, the single-hit vulnerable area [1].

But the single-hit vulnerable area is not a reliable criterion as to the vulnerability of the aircraft [2] because it cannot completely include the contribution of the multiply vulnerable components (i.e., redundant components) to the aircraft vulnerability. It is for this reason that an "equivalent" singly vulnerable area concept has been devised [2] for considering the effect of multiply vulnerable components on the vulnerability of an aircraft. Based on the expected number of hits on a vulnerable area required to kill an aircraft, the formulas for calculating the equivalent singly vulnerable area of an aircraft consisting of one or more singly vulnerable components and a set of identical multiply vulnerable components are given in [2,3]. However, the current methods can only approximately calculate the equivalent singly vulnerable area of an aircraft having several sets of identical multiply vulnerable components. Hence, the current methods cannot calculate the aircraft equivalent singly vulnerable area for the case in which 1) the multiply vulnerable components of each set do not have the same vulnerable area and 2) the aircraft vulnerable components can overlap in any arbitrary manor.

In this paper, two general methods are proposed to solve the computation of the aircraft equivalent singly vulnerable area in the aforementioned case. One is called the Monte Carlo simulation (MCS) based method, and the other is called the aircraft multiple hit vulnerability (AMHV) based method.

II. Commonly Used Methods for Calculating the Equivalent Singly Vulnerable Area

The concept of an equivalent singly vulnerable area is applicable only to impacting rounds [2,3]. This paper is based on the following assumptions: 1) a large number of hits are assumed and respective locations of the various hits on the aircraft are assumed to be taken from a uniform population; 2) a component when hit has only two states, namely, kill or nokill; 3) no commutative compound damage occurs; and 4) a kill of one component has no effect on the kill of another component.

A. One Set of Identical Multiply Vulnerable Components

The equivalent singly vulnerable area A_S for an aircraft consisting of one or more singly vulnerable components and one set of identical multiply vulnerable components, and having no overlapping components, is given by [2]

$$A_S = A_V / E_{AV} \tag{1}$$

where A_V is the summed vulnerable area of the aircraft; $A_V = (A_{v0} + nA_{v1})$, where A_{v0} is the sum of the singly vulnerable component vulnerable areas, n is the number of identical components constituting the set of multiply vulnerable components, and A_{v1} is the vulnerable area of each multiply vulnerable component; and E_{AV} is the expected number of hits on A_V required to kill the aircraft. The expression for E_{AV} in [2] is given by

$$E_{AV} = E(\alpha, n, k) = 1 + \frac{n}{(n/\alpha - 1)} + \frac{n(n-1)}{(n/\alpha - 1)(n/\alpha - 2)} + \cdots + \frac{n(n-1)\cdots(n-k+2)}{(n/\alpha - 1)(n/\alpha - 2)\cdots(n/\alpha - k + 1)} \tag{2}$$

In Eq. (2), k is the number of items in the multiply vulnerable set which must be defeated to result in the specified level of aircraft kill, and α is the fraction of the summed vulnerable area represented by the set of multiply vulnerable components

$$\alpha = (nA_{v1})/A_V \tag{3}$$

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Pei et al. [3], based on mathematical expectation theory, by simulating the kill events of identical multiply vulnerable components to a "model of filling boxes with balls," give another expression for E_{AV} as

$$E_{\mathrm{AV}} = 1 + \alpha + \alpha^2 + \cdots + \alpha^{k-2} + (1-k)\alpha^{k-1} + \sum_{i=0}^{k-2} (-1)^i C_{n-k+i}^{n-k} C_n^{n-k+1+i} \frac{k\eta_i^{k-1} - (k-1)\eta_i^k}{1-\eta_i} \tag{4}$$

where

$$\eta_i = \frac{k-i-1}{n}\alpha \tag{5}$$

Formulas (2) and (4) can give the same exact calculation results for the expected number of hits on A_V required to kill the aircraft [3].

B. More Than One Set of Identical Multiply Vulnerable Components

When an aircraft has more than one set of multiply vulnerable components, the current theoretical method can only approximately calculate the case in which the multiply vulnerable components of each set have the same vulnerable area and no overlapping exists among components. The procedure is to consider each set, one at a time, with the remaining sets considered invulnerable for the time being. The procedure is illustrated mathematically as follows [2]: Assume the ith set of multiply vulnerable components has n_i components, the vulnerable area of each multiply vulnerable component is A_{vmi}, and the number of items in the ith multiply vulnerable set which must be defeated to result in the specified level of aircraft kill is k_i. For the first set,

$$A_{S_1} = \frac{A_{v0} + n_1 A_{vm1}}{E(\alpha_1, n_1, k_1)} \tag{6}$$

where A_{S_1} is the equivalent singly vulnerable area of singly vulnerable components and the first set of multiply vulnerable components with all other sets of multiply vulnerable components considered invulnerable.

The second set of multiply vulnerable components is then introduced to obtain a new equivalent singly vulnerable area A_{S_2} where

$$A_{S_2} = \frac{A_{S_1} + n_2 A_{vm2}}{E(\alpha_2, n_2, k_2)} \tag{7}$$

and any remaining sets of multiply vulnerable components are considered invulnerable.

The procedure is similarly repeated until all significant sets of multiply vulnerable components have been considered, and the approximate aircraft total equivalent singly vulnerable area A_S can be finally obtained.

III. Monte Carlo Simulation Based Method

The flowchart of the proposed MCS based method for calculating the aircraft equivalent singly vulnerable area is shown in Fig. 1. More details about the flowchart are presented in the following.

A. Analysis of the Aircraft Unique Existent States

Pei and Song [4] point out that aircraft hit by a threat can exist in several unique states. For example, a sample aircraft consists of four critical components: pilot, fuel tank, engine 1, and engine 2. The pilot and the fuel tank are singly vulnerable components, and engine 1 and engine 2 constitute the only set of multiply vulnerable components. The example aircraft can exist in the four unique states: 1) the aircraft has been killed, 2) only engine 1 has been killed, 3) only engine 2 has been killed, and 4) none of the critical components has been killed.

In this paper, we assume that the aircraft exists in $s+2$ unique states. The first state refers to the state in which any singly vulnerable component or any set of multiply vulnerable components is killed, and for ease of illustration we call it the "kill state." The second state

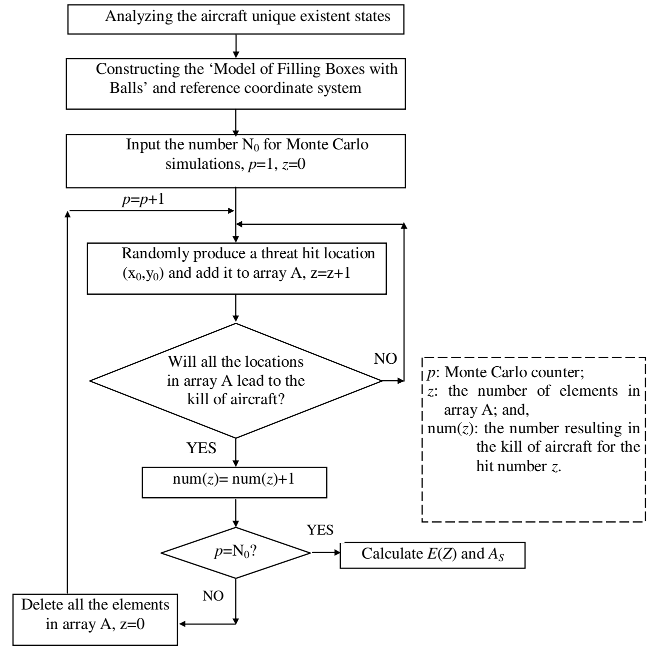

Fig. 1 Flowchart for calculating the equivalent singly vulnerable area using the MCS method.

refers to the state in which none of the critical components is killed and we call it the "nonkill state." The last s states refer to the states of the multiply vulnerable components and the combinatorial states among them (except the "kill state" and the "nonkill state"), and we call the s states "redundant states." No matter whether components overlap or not for a given threat aspect, an aircraft hit by a threat could exist in the three kinds of states: the kill state, the nonkill state, and redundant states. More details about determining the aircraft's unique existent states and the areas corresponding to each state can be found in [5].

B. Constructing the Model of Filling Boxes with Balls and Reference Coordinate System

Figure 2 shows the model of filling boxes with balls [3] for aircraft equivalent singly vulnerable area calculation. The upper part of the figure is corresponding to the kill state, and the lower part of the figure is corresponding to redundant states.

Let h and w be the height and the width of the model, respectively; h_0 be the height of the box corresponding to the vulnerable area of the kill state; h_1 be the height of the boxes corresponding to the vulnerable areas of redundant states; A_{Vi} and w_i be the vulnerable area and the associated width of the box corresponding to the ith redundant state, respectively; and A_{VK} be the vulnerable area of the kill state. Then

$$A_V = A_{\mathrm{VK}} + \sum_{i=1}^{s} A_{Vi} \tag{8}$$

Let

$$w = h = \sqrt{A_V} \tag{9}$$

Hence,

$$h_0 = A_{\mathrm{VK}}/w \tag{10}$$

$$h_1 = \left(\sum_{i=1}^{s} A_{Vi}\right)/w \tag{11}$$

$$w_i = A_{Vi}/h_1 \qquad (i = 1, 2, 3, \ldots, s) \tag{12}$$

To produce the random hit locations, which are two-dimensionally and uniformly distributed in rectangle OABC of Fig. 2, for

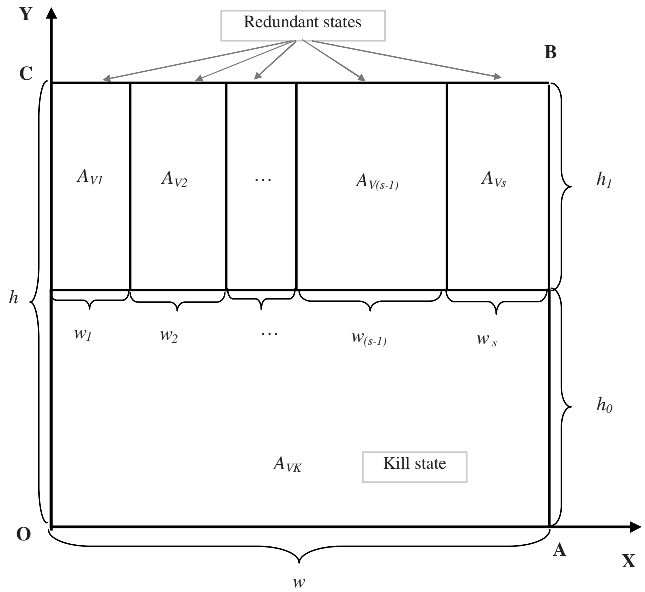

Fig. 2 The model of filling boxes with balls [3] for equivalent singly vulnerable area calculation.

Monte Carlo simulations, we should build the reference coordinate system XOY. The selection of the coordinate origin and the coordinate axis depends on the ease of calculation, and we choose the coordinate system shown in Fig. 2 for illustration of our proposed method.

C. Criterion for the Kill of a Component

According to the model of filling boxes with balls [3], when the box contains the random hit location (x_0, y_0), the existent state corresponding to that box will happen implying the component(s) corresponding to that state will be killed. Hence, once knowing the state happens or not, we can determine whether the component(s) is (are) killed or not. The criterion for the occurrence of the kill state is that the location (x_0, y_0) satisfies

$$0 < x_0 < w \tag{13}$$

and

$$0 < y_0 < h_0 \tag{14}$$

The criterion for the occurrence of the ith redundant state, whose vulnerable area is A_{Vi}, is that the location (x_0, y_0) satisfies

$$\sum_{r=0}^{i-1} w_r < x_0 < \sum_{t=0}^{i} w_t \tag{15}$$

and

$$h_0 < y_0 < h_0 + h_1 \tag{16}$$

where

$$w_0 = 0 \tag{17}$$

D. Criterion for the Kill of Aircraft

An aircraft kill can be determined by the states of the singly vulnerable components and the sets of multiply vulnerable components. It means that the kill of any of the singly vulnerable components or any set of the multiply vulnerable components can lead to the kill of aircraft. The criterion for the kill of singly vulnerable components is given by formulas (13) and (14). The criterion for the kill of the ith set of multiply vulnerable components is that the number of items in the multiply vulnerable set that has been defeated is equal to or more than k_i. The criterion for the kills of each multiply vulnerable component can be determined by formulas (15–17).

E. Calculating the Aircraft Equivalent Singly Vulnerable Area Using Monte Carlo Simulations

In the actual aircraft combat operation, aircraft may be killed after one threat hit on A_V or after two threat hits, or after z threat hits. According to the original definition of mathematical expectation [6], we have

$$\begin{aligned} E_{\mathrm{AV}} &= E(Z) = 1\times P(z=1) + 2\times P(z=2) + 3\times P(z=3) + \cdots \\ &= \sum_{z=1}^{+\infty} zP(Z=z) \end{aligned} \tag{18}$$

where $P(Z=z)$ denotes the appearance probability for the random variable $Z=z$, $z=1,2,\ldots,+\infty$. After the determination of E_{AV}, the aircraft equivalent singly vulnerable area can be attained by formula (1). The procedure for calculating E_{AV} using the Monte Carlo simulations is illustrated mathematically as follows.

As is shown in Fig. 1, $\mathrm{num}(z)$ denotes the number resulting in the kill of the aircraft for the hit number z in the N_0 Monte Carlo simulations. When the simulation number N_0 is sufficient enough, $P(Z=z)$ can be approximately determined by

$$P(Z=z) \approx \mathrm{num}(z)/N_0 \tag{19}$$

Hence,

$$E_{\mathrm{AV}} = \sum_{z=1}^{+\infty} zP(Z=z) \approx \sum_{z=1}^{+\infty} z\times \mathrm{num}(z)/N_0 \tag{20}$$

and

$$A_S \approx A_V / \sum_{z=1}^{+\infty} z\times \mathrm{num}(z)/N_0 \tag{21}$$

IV. Aircraft Multiple Hit Vulnerability Based Method

The AMHV based method for calculating the aircraft equivalent singly vulnerable area is illustrated as follows.

Let $\bar{P}^N_{K/H}$ be the aircraft's cumulated probability of kill after N threat hits, and let E_{Ap} be the expected number of hits on aircraft presented area A_P required to kill the aircraft. Because the threat hit locations are uniformly distributed on A_p, we have

$$E_{\mathrm{AV}} = E_{Ap}(A_V/A_P) \tag{22}$$

A Boolean variable Y_N is introduced to indicate the aircraft kill and nokill state after N threat hits

$$Y_N = \begin{cases} 1 & \text{aircraft survives after } N \text{ hits} \\ 0 & \text{aircraft is killed after } N \text{ hits} \end{cases} \tag{23}$$

Then

$$Y = \sum_{N=0}^{+\infty} Y_N = Y_0 + Y_1 + Y_2 + \cdots \tag{24}$$

denotes the total hit number when the aircraft is killed. Because

$$P(Y_N)\begin{cases} 1-\bar{P}^N_{K/H} & Y_N = 1 \\ \bar{P}^N_{K/H} & Y_N = 0 \end{cases} \tag{25}$$

and [6]

$$\begin{aligned} E_{Ap} &= E(Y) = E(Y_0 + Y_1 + Y_2 + \cdots) \\ &= E(Y_0) + E(Y_1) + E(Y_2) + \cdots \end{aligned} \tag{26}$$

we obtain

$$E_{Ap} = \sum_{N=0}^{+\infty} 1\times(1-\bar{P}^N_{K/H}) + 0\times \bar{P}^N_{K/H} = \sum_{N=0}^{+\infty}(1-\bar{P}^N_{K/H}) \tag{27}$$

Noting that $\bar{P}^N_{K/H}$ satisfies 1) $\bar{P}^0_{K/H} = 0$, 2) $\bar{P}^{N+1}_{K/H} \geq \bar{P}^N_{K/H}$, and 3) $\bar{P}^{+\infty}_{K/H} = 1$, formula (27) can be rewritten as

$$E_{Ap} = \sum_{N=0}^{N_{\text{end}}} (1 - \bar{P}^N_{K/H}) \tag{28}$$

where N_{end} is determined by

$$1 - \bar{P}^{N_{\text{end}}}_{K/H} < \varepsilon \tag{29}$$

In Eq. (29), ε is a very small positive number. Substituting Eq. (28) into Eq. (22) leads to

$$E_{\text{AV}} = \left[\sum_{N=0}^{N_{\text{end}}} (1 - \bar{P}^N_{K/H})\right](A_V/A_P) \tag{30}$$

In sum, the AMHV based method for calculating the equivalent singly vulnerable area can be generalized into three steps: 1) Referring to [4,5], calculate the cumulated probability of the kill of an aircraft that may have several sets of different multiply vulnerable components and may have overlapping vulnerable components; 2) calculate the E_{A_p} by Eqs. (28) and (29); and 3) calculate E_{AV} and A_S by Eq. (30) and Eq. (1), respectively.

V. Examples

In this section, four examples are provided for verifying the correctness of the two proposed general methods, the MCS based method and the AMHV based method, for calculating the aircraft equivalent singly vulnerable area.

A. Example 1

As mentioned previously, there is only the exact calculation method, described in Sec. II.A, for calculating the equivalent singly vulnerable area for an aircraft consisting of one set of identical multiply vulnerable components, and so we will validate the correctness of the proposed methods using three cases of aircraft consisting of several singly vulnerable components and one set of identical multiply vulnerable components. Table 1 presents the original calculation data and the comparison of the results from the exact method, the MCS method, and the AMHV method.

Table 1 shows that, the calculation result using the MCS method or the AMHV method approaches the exact result very well, and the relative error is within 1%.

B. Example 2

The approximate calculation method, described in Sec. II.B, for calculating the equivalent singly vulnerable area of aircraft having more than one set of identical multiply vulnerable components can roughly validate the correctness of the proposed methods. Table 2 presents the original calculation data and the comparison of the results from the approximate method, the MCS method, and the AMHV method for three cases of aircraft. In addition, the presented area A_P of each aircraft is 1000 ft^2.

Table 2 shows that the calculation results using the MCS method and the AMHV method are in agreement with the approximate result and the relative errors are within 2% and 1%, respectively.

C. Example 3

The following presents the computation examples for three cases of the equivalent singly vulnerable area of an aircraft having more than one set of different multiply vulnerable components and having no overlapping components. For comparison purposes, we use some original data of case 3 in Table 2 for our presentation, and we assume 1) the multiply vulnerable components of each set do not always have the same vulnerable area and 2) each set of multiply vulnerable components of the three cases has the same total vulnerable area. Table 2 presents the original calculation data and the comparison of the results from the MCS method and the AMHV method, where A_{vij} denotes the jth component vulnerable area in the ith set of multiply vulnerable components.

Table 3 implies that each set of multiply vulnerable components of the three cases has the same total vulnerable area; however, the aircraft equivalent singly vulnerable areas of the three cases are totally different because the multiply vulnerable components of each set do not always have the same vulnerable area. Moreover, the relative error between the results from the MCS method and from the AMHV method is within 1%.

D. Example 4

This example is to calculate the equivalent singly vulnerable area of an aircraft having two sets of different multiply vulnerable

Table 1 Comparison of A_S, ft^2, for aircraft having one set of identical vulnerable components

							MCS			AMHV	
Case	A_{v1}/ft^2	A_{v0}/ft^2	A_P/ft^2	n	k	Exact result	$N_0 = 5000$	$N_0 = 10,000$	$N_0 = 100,000$	$\varepsilon = 10^{-3}$	$\varepsilon = 10^{-4}$
1 [2]	10.0	40.0	400.0	2	2	42.857	42.934	43.253	42.876	42.886	42.860
2	5.0	80.0	400.0	3	2	81.429	81.489	81.819	81.448	81.479	81.434
3	25.0	100.0	400.0	4	2	127.273	127.551	126.097	127.101	127.306	127.276

Table 2 Comparison of A_S, ft^2, for aircraft having several sets of identical vulnerable components

			MCS			AMHV	
Case	Original data	Approximate result	$N_0 = 5000$	$N_0 = 10{,}000$	$N_0 = 100{,}000$	$\varepsilon = 10^{-3}$	$\varepsilon = 10^{-4}$
1 [2]	Two sets of identical multiply vulnerable components: $A_{v0} = 100$ ft^2; $A_{vm1} = 25$ ft^2, $n_1 = 4$, $k_1 = 2$; $A_{vm2} = 2$ ft^2, $n_2 = 2$, $k_2 = 2$.	127.500	126.520	127.868	127.581	127.396	127.326
2	Two sets of identical multiply vulnerable components: $A_{v0} = 100$ ft^2; $A_{vm1} = 20$ ft^2, $n_1 = 4$, $k_1 = 2$; $A_{vm2} = 10$ ft^2, $n_2 = 2$, $k_2 = 2$.	121.333	122.130	121.286	121.201	121.174	121.107
3	Four sets of identical multiply vulnerable components: $A_{v0} = 80$ ft^2; $A_{vm1} = 10$ ft^2, $n_1 = 3$, $k_1 = 3$; $A_{vm2} = 15$ ft^2, $n_2 = 2$, $k_2 = 2$; $A_{vm3} = 9$ ft^2, $n_3 = 3$, $k_3 = 2$; $A_{vm4} = 20$ ft^2, $n_4 = 2$, $k_4 = 2$.	93.250	91.935	92.164	92.275	92.445	92.396

Table 3 Comparison of A_S, ft^2, for aircraft having several sets of different vulnerable components and having no overlapping components

Case	Original data (see Table 2 for other data)	MCS			AMHV	
		$N_0 = 5000$	$N_0 = 10,000$	$N_0 = 100,000$	$\varepsilon = 10^{-3}$	$\varepsilon = 10^{-4}$
1 (case 3 in Table 2)	$A_{v11} = 10$ ft^2, $A_{v12} = 10$ ft^2, $A_{v13} = 10$ ft^2; $A_{v21} = 15$ ft^2, $A_{v22} = 15$ ft^2; $A_{v31} = 9$ ft^2, $A_{v32} = 9$ ft^2, $A_{v33} = 9$ ft^2; $A_{v41} = 20$ ft^2, $A_{v42} = 20$ ft^2.	91.935	92.164	92.275	92.445	92.396
2	$A_{v11} = 5$ ft^2, $A_{v12} = 10$ ft^2, $A_{v13} = 15$ ft^2; $A_{v21} = 10$ ft^2, $A_{v22} = 20$ ft^2; $A_{v31} = 5$ ft^2, $A_{v32} = 5$ ft^2, $A_{v33} = 17$ ft^2; $A_{v41} = 30$ ft^2, $A_{v42} = 10$ ft^2.	89.556	90.314	90.412	90.158	90.107
3	$A_{v11} = 5$ ft^2, $A_{v12} = 5$ ft^2, $A_{v13} = 20$ ft^2; $A_{v21} = 5$ ft^2, $A_{v22} = 25$ ft^2; $A_{v31} = 5$ ft^2, $A_{v32} = 5$ ft^2, $A_{v33} = 17$ ft^2; $A_{v41} = 5$ ft^2, $A_{v42} = 35$ ft^2.	87.831	87.969	87.434	87.452	87.393

Table 4 Aircraft unique existent states and corresponding vulnerable areas [5]

State	Kill event	Corresponding area/ft^2
A_{VK}	Aircraft killed: $A \cup B \cup C \cup D \cup EF \cup GH$	29.97423
A_{V1}	Only component e killed: $E \cap \bar{F} \cap \bar{G} \cap \bar{H} \cap \bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}$	1.58242
A_{V2}	Only component f killed: $\bar{E} \cap F \cap \bar{G} \cap \bar{H} \cap \bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}$	1.29471
A_{V3}	Only component g killed: $\bar{E} \cap \bar{F} \cap G \cap \bar{H} \cap \bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}$	0.08702
A_{V4}	Only component h killed: $\bar{E} \cap \bar{F} \cap \bar{G} \cap H \cap \bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}$	0.06926
A_{V5}	Only components e and g killed: $E \cap \bar{F} \cap G \cap \bar{H} \cap \bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}$	0.08702
A_{V6}	Only components e and h killed: $E \cap \bar{F} \cap \bar{G} \cap H \cap \bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}$	0.06926
A_{V7}	Only components f and g killed: $\bar{E} \cap F \cap G \cap \bar{H} \cap \bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}$	0.07120
A_{V8}	Only components f and h killed: $\bar{E} \cap F \cap \bar{G} \cap H \cap \bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}$	0.06926
Nonkill state	None of the critical components killed	0.07120

components and having overlapping components. The aircraft contains eight critical components. Components a, b, c, and d are singly vulnerable components, components e and f constitute the 1st set of multiply vulnerable components, and components g and h constitute the 2nd set of multiply vulnerable components. At the given threat aspect, there exists component overlapping, and more details about the description of overlapping regions can be found in [5]. After vulnerable area decomposition in the overlapping regions mentioned in [5], the aircraft can exist in 10 distinct states listed in Table 4.

The example aircraft equivalent singly vulnerable areas using the MCS method are 2.23119 for $N_0 = 5000$, 2.23531 for $N_0 = 10,000$, and 2.25266 for $N_0 = 100{,}000$; the example aircraft equivalent singly vulnerable areas using the AMHV method are 2.25081 for $\varepsilon = 10^{-3}$ and 2.25058 for $\varepsilon = 10^{-4}$.

The preceding data show that the A_S result using the MCS method is in good agreement with that from the AMHV method, and the relative error is within 1%.

VI. Conclusions

The preceding four examples show the correctness of the two proposed general methods, the MCS based method and the AMHV based method (AMHV), for calculating the aircraft equivalent singly vulnerable area. The two methods outlined provide very similar accuracy compared to the present method (for components with a single item). It should be mentioned that, with respect to the computation efforts and coding "intensity," the MCS method is relatively better than the AMHV method.

The MCS method is based on an aircraft's unique existent states and the vulnerable areas corresponding to each existent state. The analysis of the aircraft's unique existent states can be found in [5], which uses the "aircraft vulnerable area decomposition method" to analyze the aircraft existent states in the overlapping region of vulnerable components.

The AMHV method is based on the cumulated probability of kill for an aircraft subjected to multiple threat hits. The multiple hit vulnerability calculating methods for an aircraft that may have overlapping components and multiply vulnerable components can be found in [4,5].

The two methods are very general and can calculate the equivalent singly vulnerable area of an aircraft that may have several sets of different multiply vulnerable components and have overlapping components.

The aircraft equivalent singly vulnerable area can totally consider the contribution of multiply vulnerable components to aircraft vulnerability. Hence, three applications of the concept of aircraft equivalent singly vulnerable area are recommended: 1) compare the relative vulnerability of different aircrafts, 2) compare the relative vulnerability of the same aircraft at different threat hit aspects, and 3) improve the Poisson approach for calculating the aircraft's probability of kill from missile fragments.

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